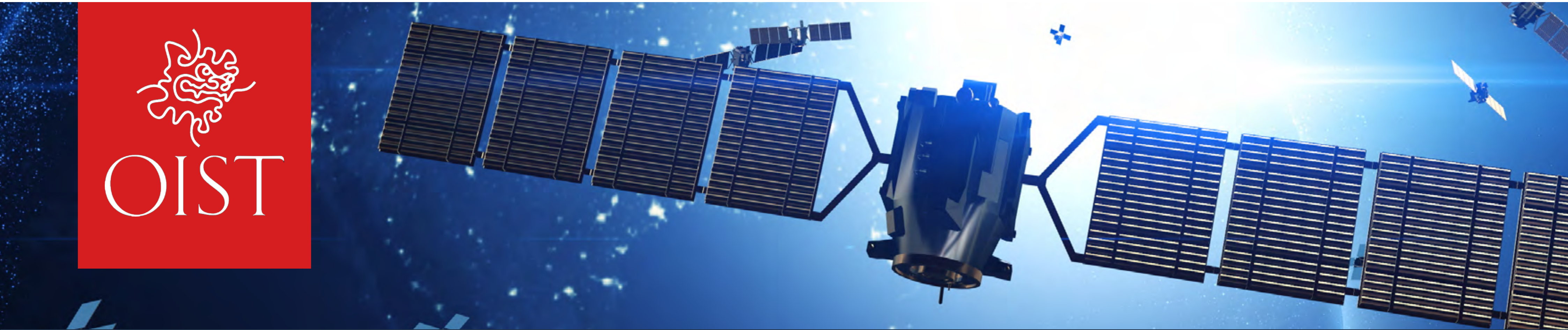


Lecture 3: Advanced Quantum Repeaters

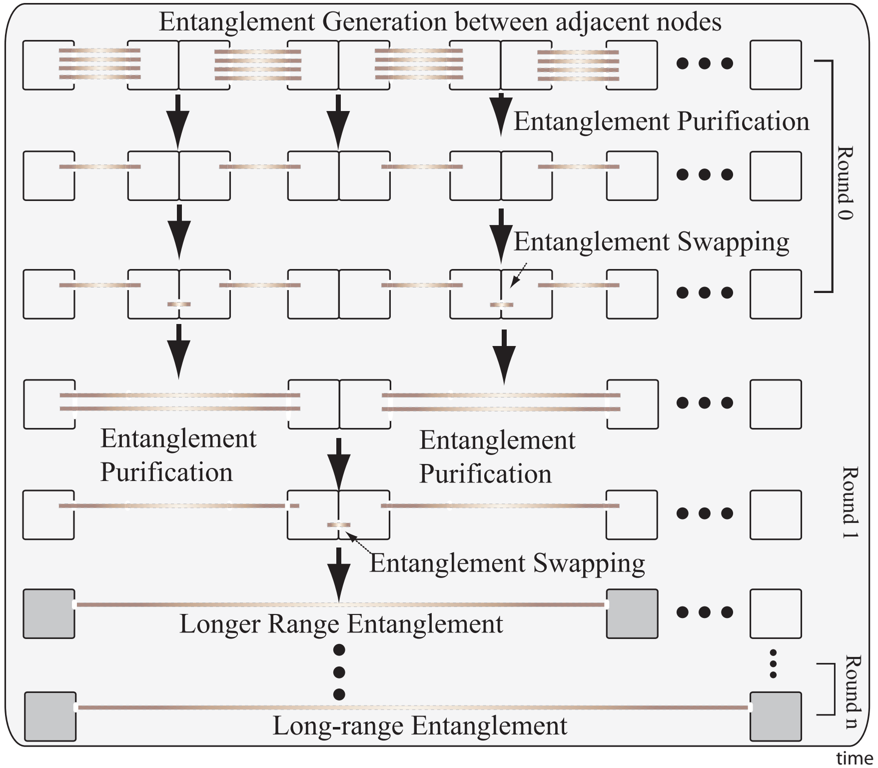
Bill Munro

email: bill.munro@oist.jp

Overview

- Discuss limitations in first generation quantum repeaters
- Introduce the concept of second and third generation quantum repeaters
- Introduce quantum error correction
- Discuss performance of second generation quantum repeaters

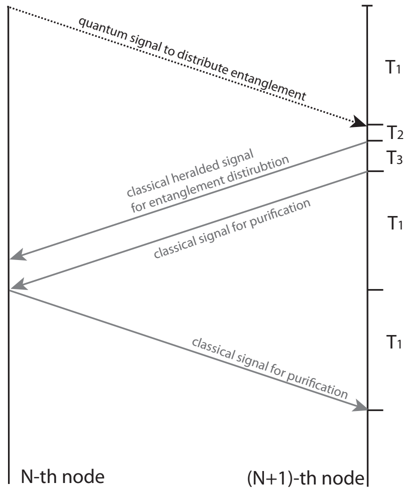
Limitations of the first generation QRs

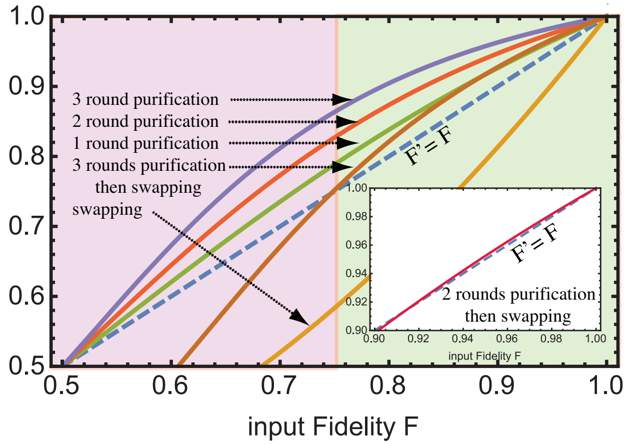


- Channel losses mean sending the photon is probabilistic!!!!

To recap

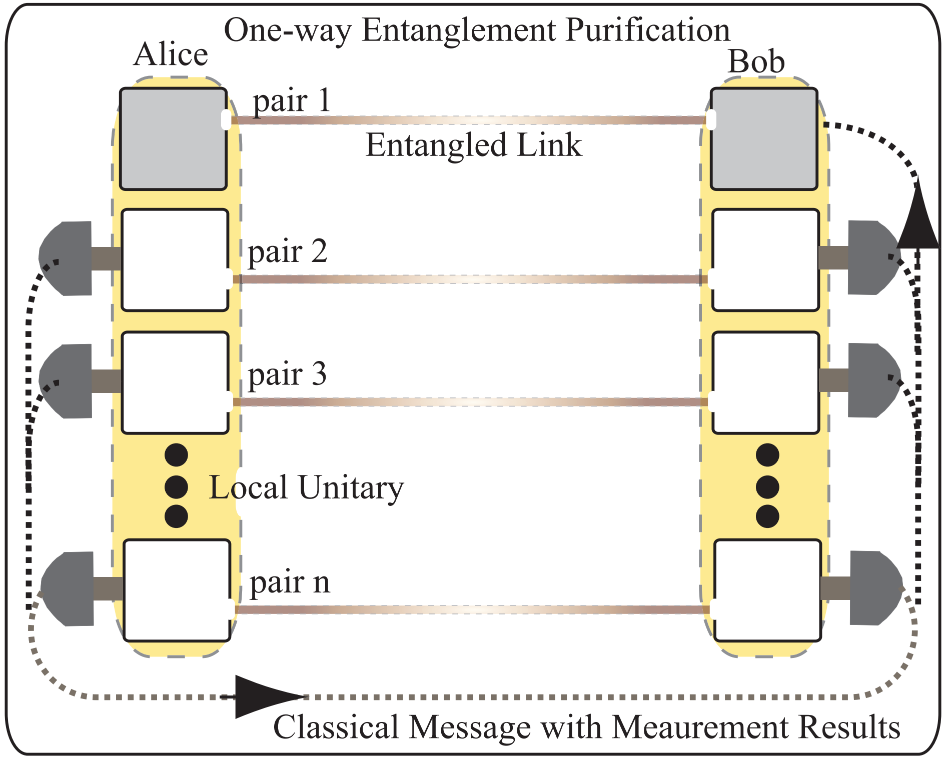
- In our earlier lecture on first generation QR we showed the necessity including entangelment purification as one of the operations
- Entanglement purification is probabilistic but heralded and occurs at many distances.
- Need a deterministic operation





One Step Quantum Purification

- The problem with most purification protocols is that there are potentially many rounds of purification required.
- We can think about using quantum error detection codes (still probabilistic but reduces the number of purification rounds)

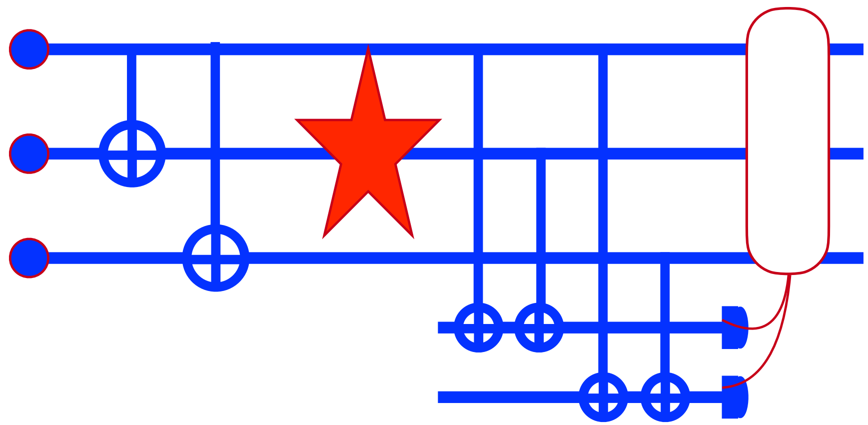


An alternative is quantum error correction but comes at a huge cost!!

Saves waiting for classical communication however!!!

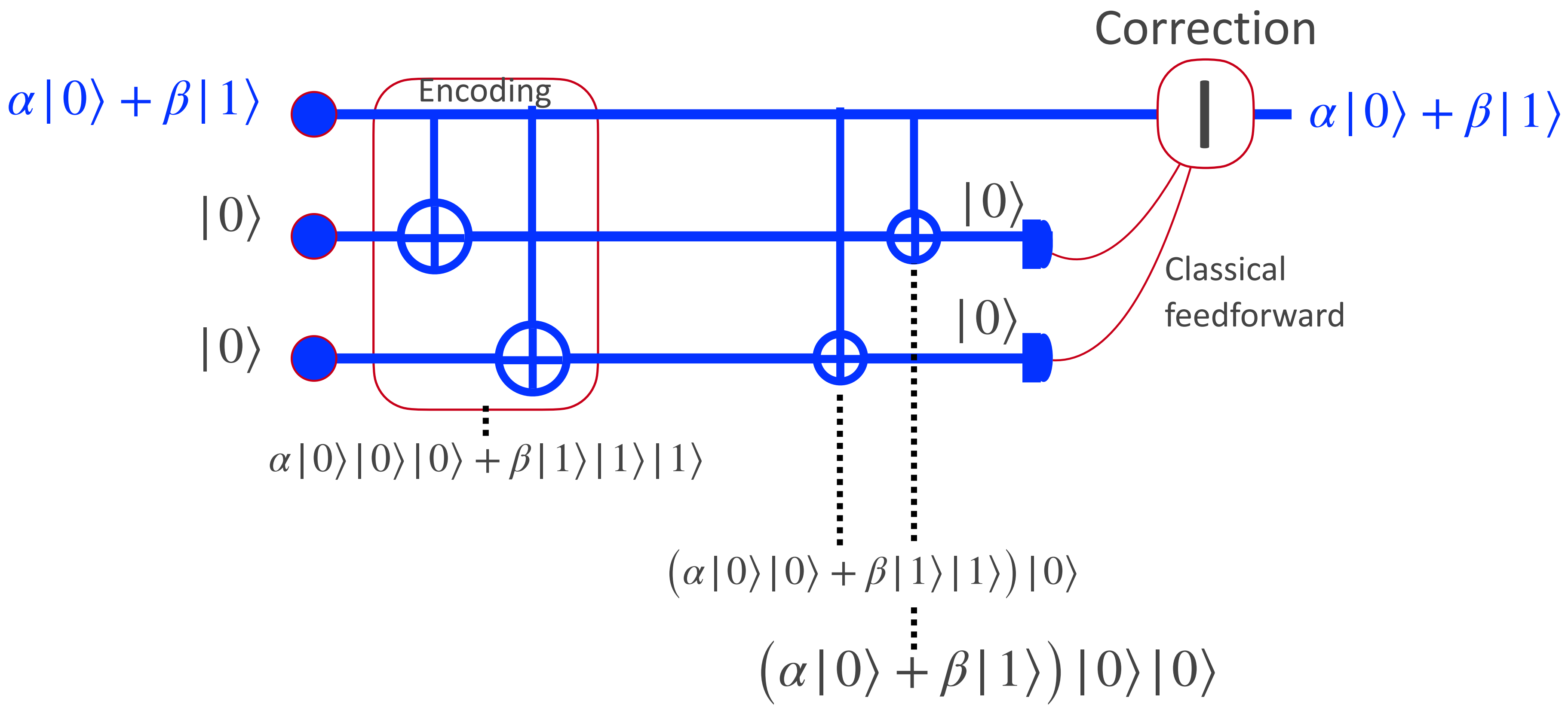
Quantum error correction

- It is an algorithm that helps correct errors in our computation/communication
- The three qubit code is the simplest



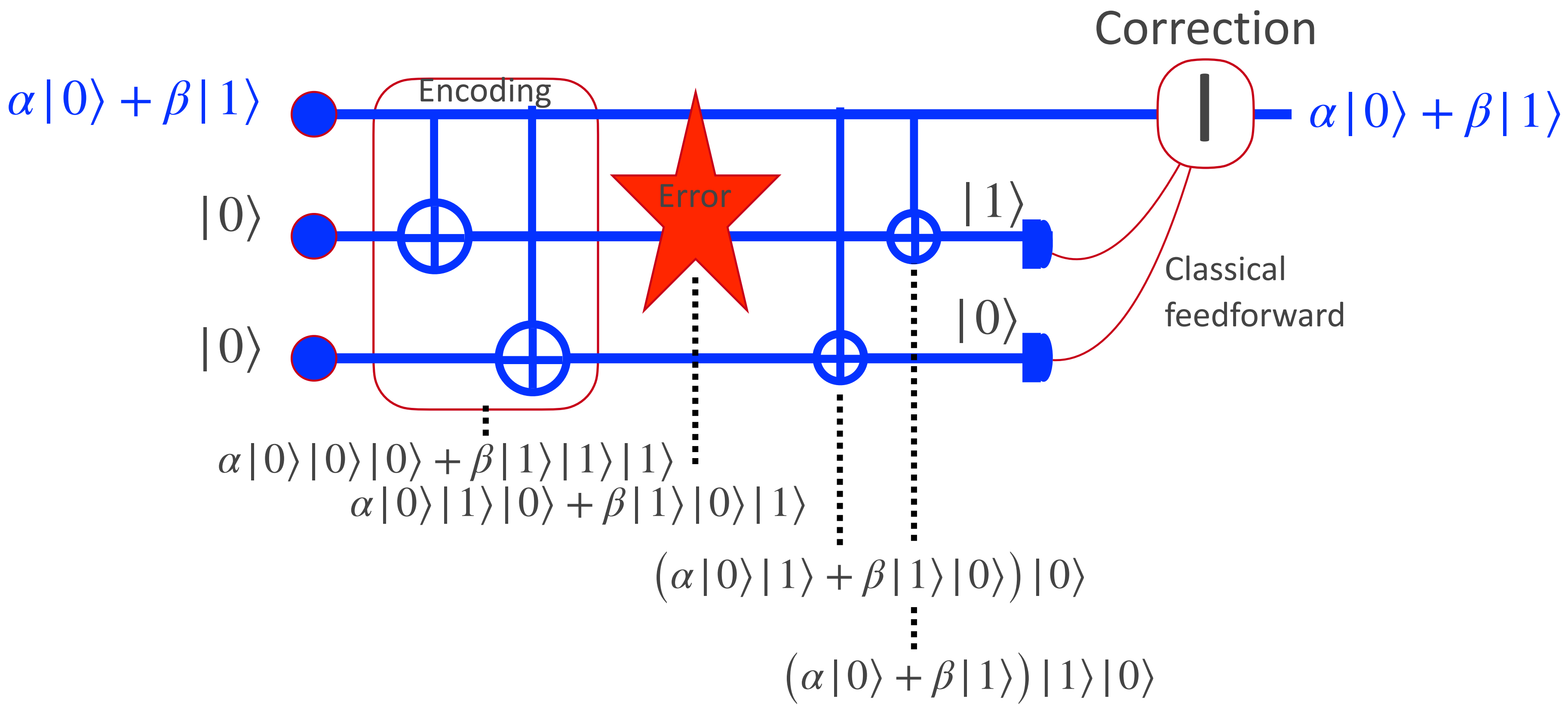
The Bit flip code (Simplified)

- Lets see how it works (no error)



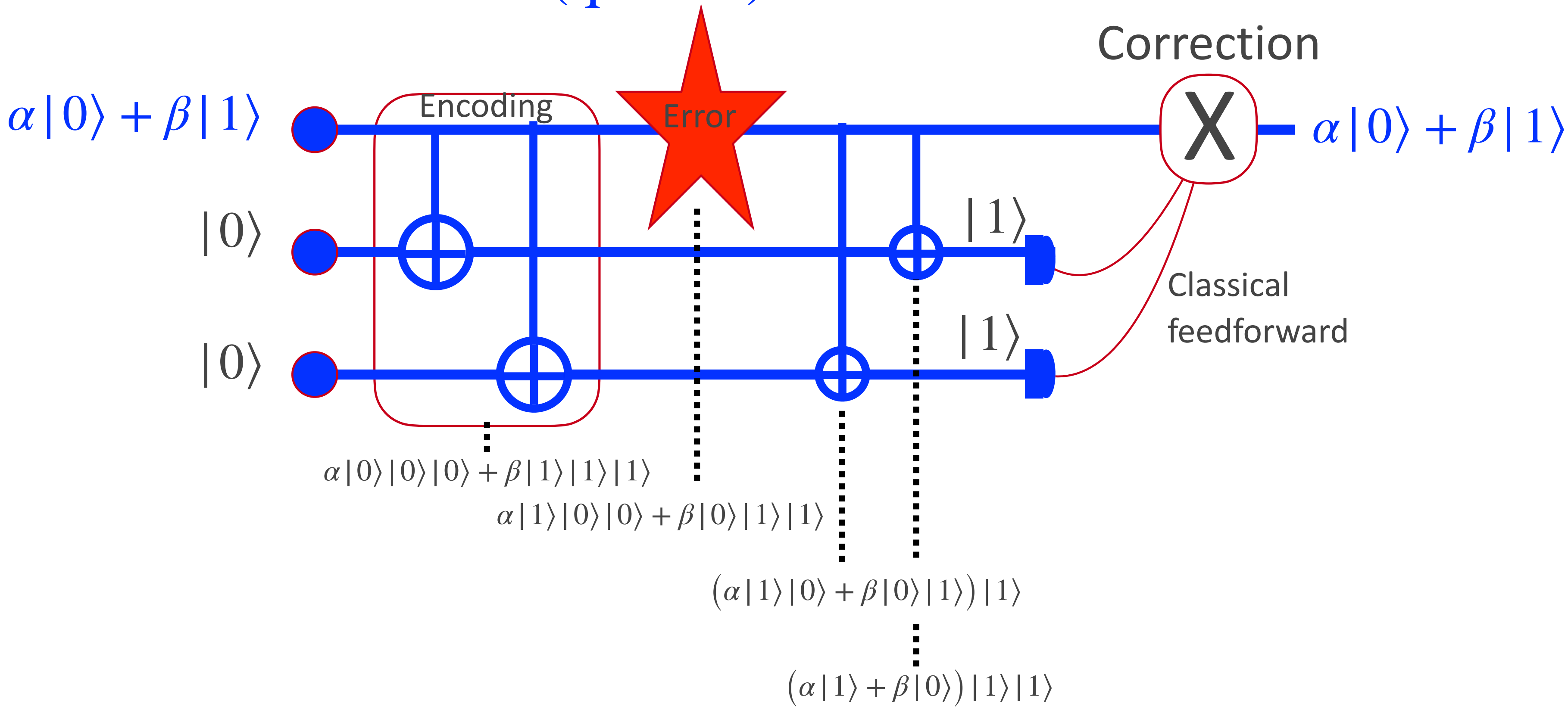
The Bit flip code (Simplified)

- Lets see how it works (qubit 2)



The Bit flip code (Simplified)

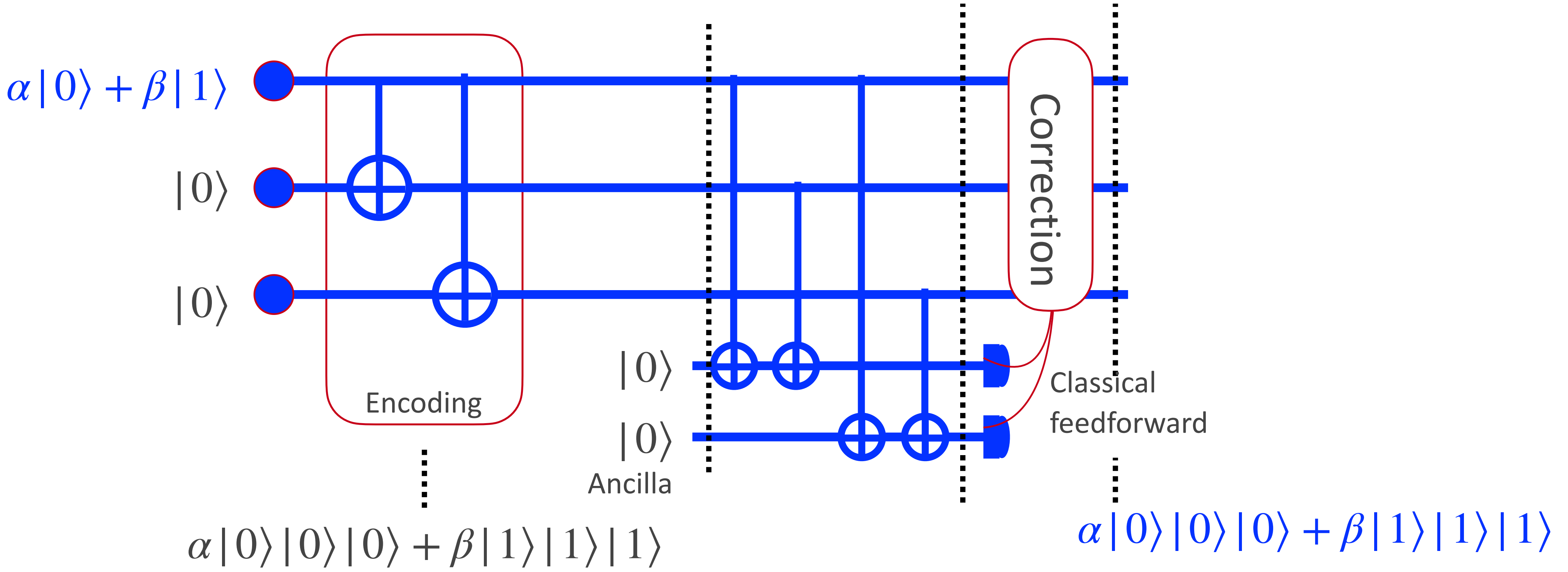
- Lets see how it works (qubit 1)



- In error correction - do not decode the qubits

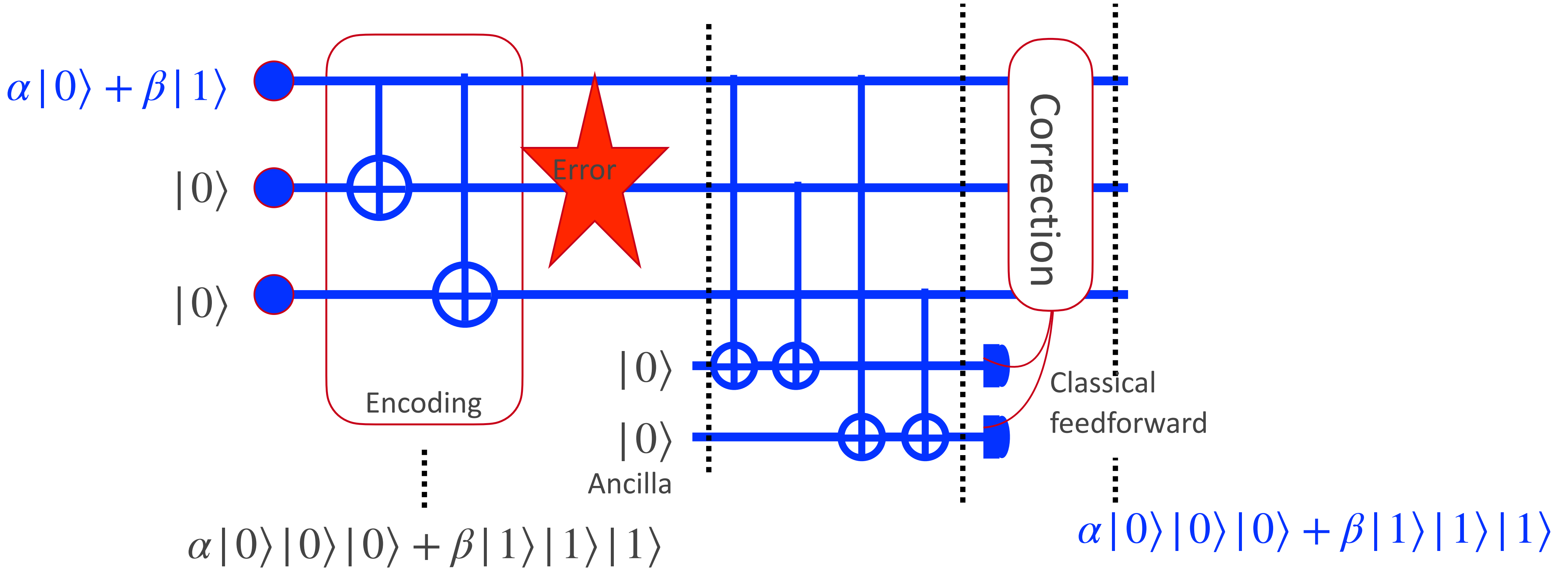
The Bit flip code

- Lets see how it works



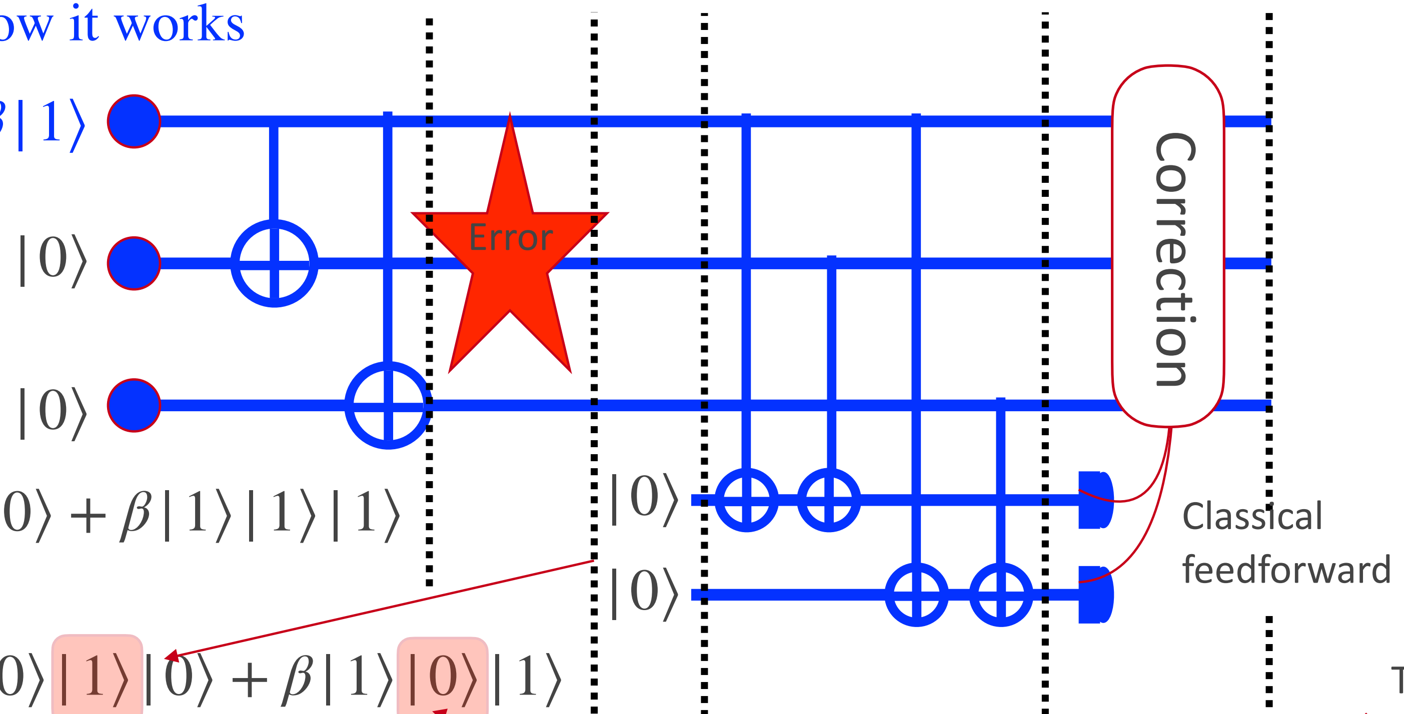
The Bit flip code

- Lets see how it works



The Bit flip code

- Lets see how it works



$\alpha|0\rangle|0\rangle|0\rangle+\beta|1\rangle|1\rangle|1\rangle$

$\alpha|0\rangle|1\rangle|0\rangle+\beta|1\rangle|0\rangle|1\rangle$

Error

$[\alpha|0\rangle|1\rangle|0\rangle+\beta|1\rangle|0\rangle|1\rangle]|0\rangle|0\rangle$ Ancilla

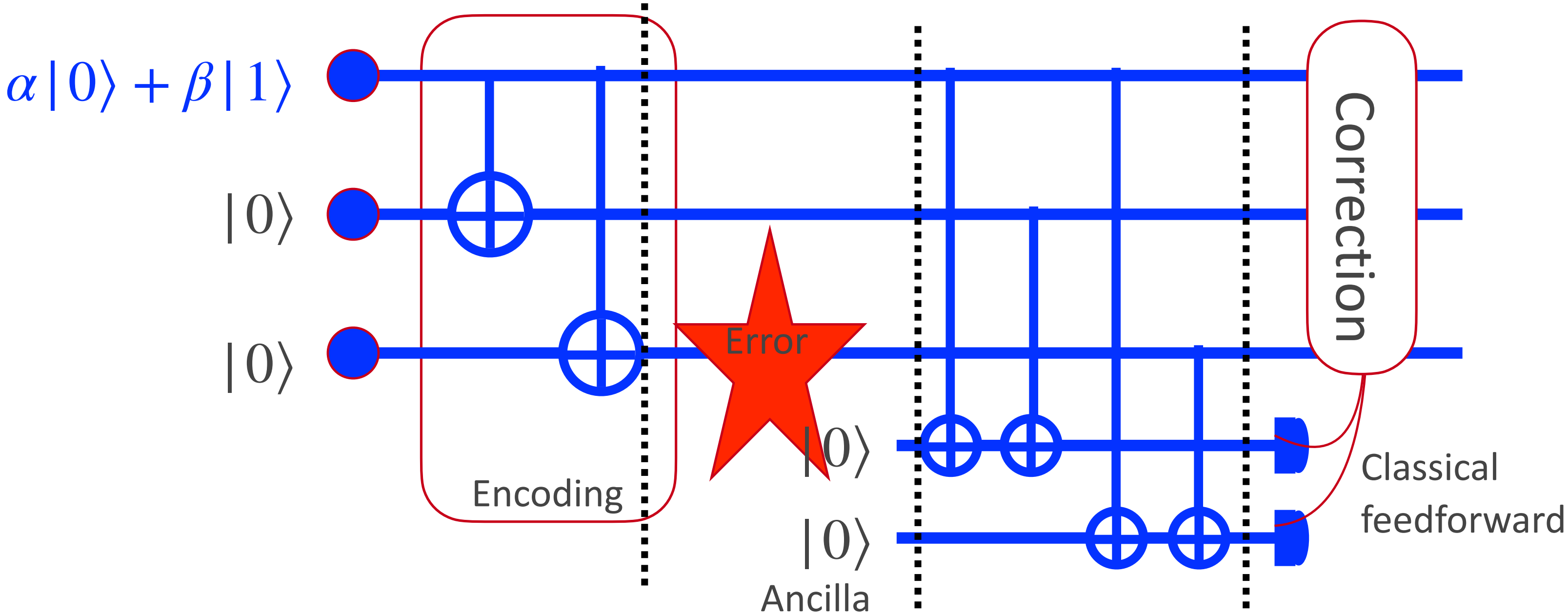
Tell us to [erform a bit flip on qubit 2

$\alpha|0\rangle|1\rangle|0\rangle|1\rangle|0\rangle+\beta|1\rangle|0\rangle|1\rangle|1\rangle|0\rangle=\{\alpha|0\rangle|1\rangle|0\rangle+\beta|1\rangle|0\rangle|1\rangle\}|1\rangle|0\rangle$

$\alpha|0\rangle|0\rangle|0\rangle+\beta|1\rangle|1\rangle|1\rangle$

The Bit flip code

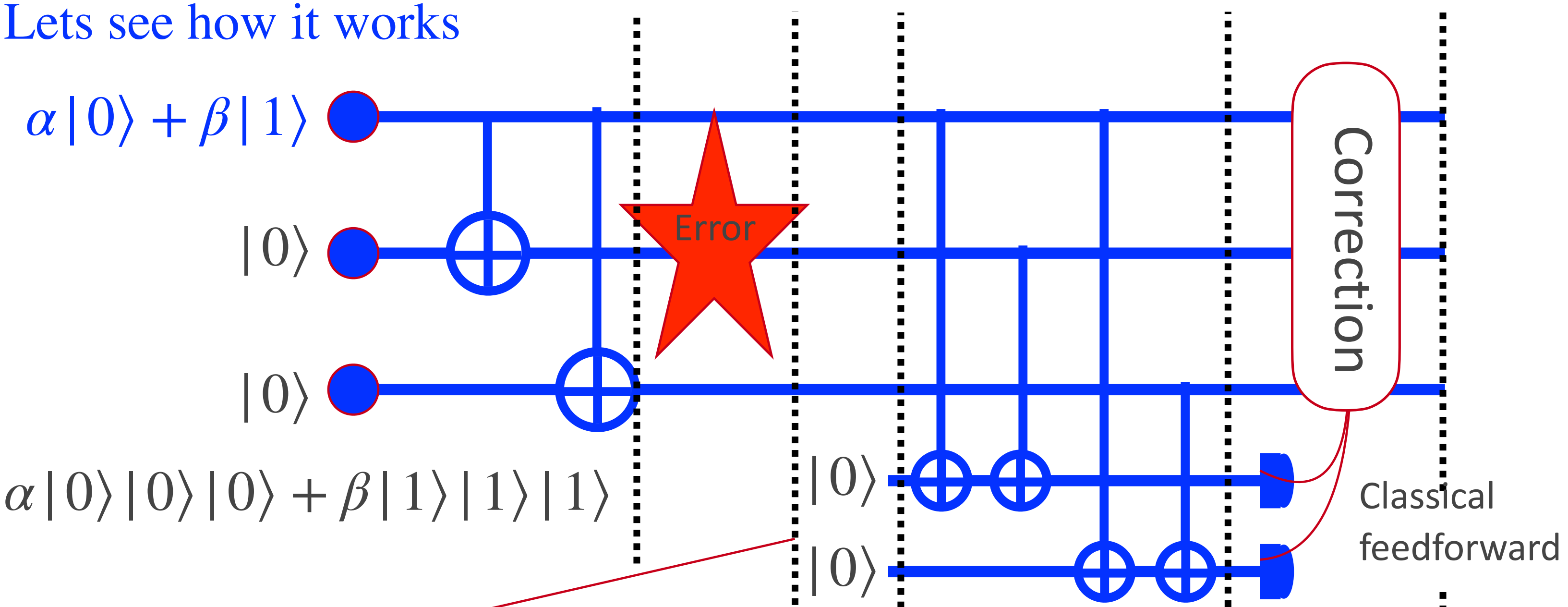
- Consider an error on qubit 3



- Work out the quantum state at each dashed line
- Determine the state of the two ancilla qubits
- What is the correction operation?

The Bit flip code

- Lets see how it works



$\alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle$

$\alpha|0\rangle|1\rangle|0\rangle + \beta|1\rangle|0\rangle|1\rangle$

Error

$[\alpha|0\rangle|1\rangle|0\rangle + \beta|1\rangle|0\rangle|1\rangle]|0\rangle|0\rangle$ Ancilla

$\alpha|0\rangle|1\rangle|0\rangle|1\rangle|0\rangle + \beta|1\rangle|0\rangle|1\rangle|1\rangle|0\rangle = \{\alpha|0\rangle|1\rangle|0\rangle + \beta|1\rangle|0\rangle|1\rangle\}|1\rangle|0\rangle$

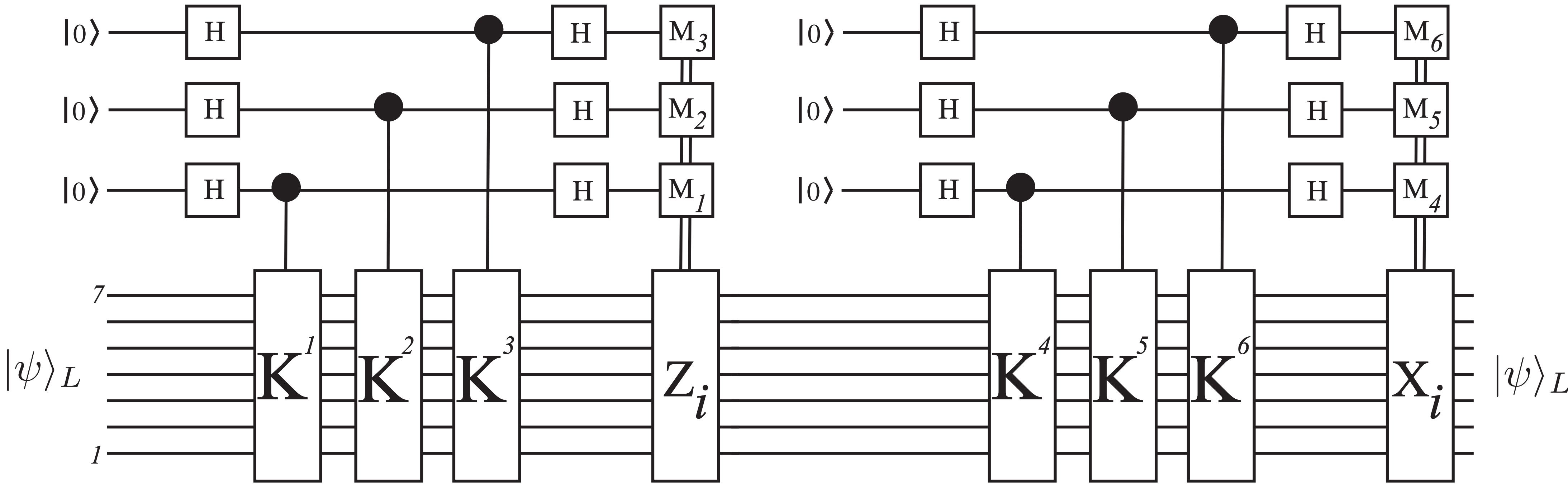
Tell us what error occurred with measuring qubit 1, 2, 3

$|0\rangle|0\rangle \to I$
$|0\rangle|1\rangle \to X_3$
$|1\rangle|0\rangle \to X_2$
$|1\rangle|1\rangle \to X_1$

$\alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle$

The 7 qubit Steane code

- Correct an arbitrary single qubit error (X, Z, XZ)



$$i = 1^{M_2} + 2^{M_3} + 4^{M_1} \qquad i = 1^{M_5} + 2^{M_6} + 4^{M_4}$$

Steane A M 1996 Error correcting codes in quantum theory Phys. Rev. Lett. **77** 793

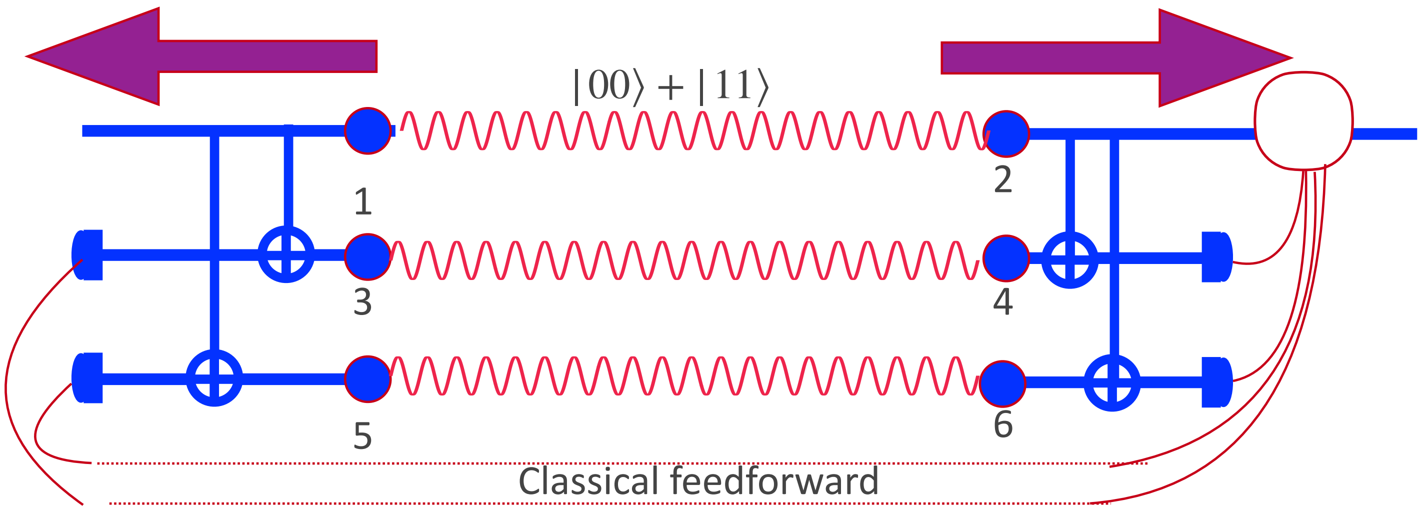
How do this help in communication?

Do we need to send encoded qubits?

$$|\Phi_{+}\rangle = |00\rangle + |11\rangle \qquad |\Psi_{+}\rangle = |01\rangle + |10\rangle$$

The Bit flip code for communication

- How do we get this to work for communication?



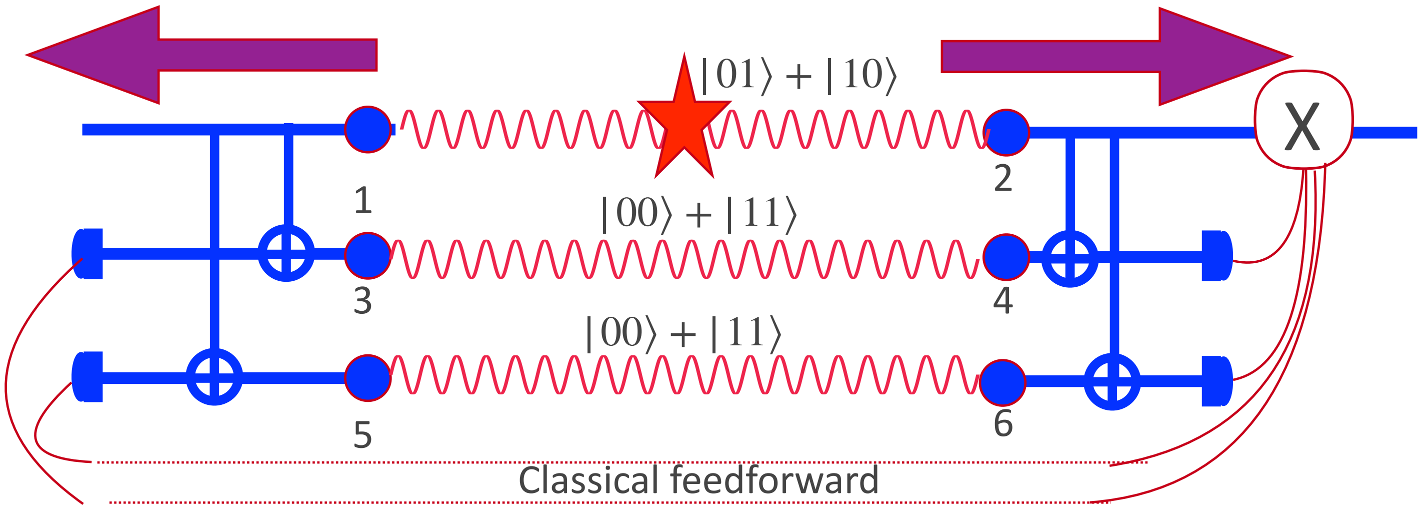
- Look at the ideal case $(|00\rangle + |11\rangle)(|00\rangle + |11\rangle)(|00\rangle + |11\rangle)$

$$
\begin{array}{l}
000000 \rightarrow 000000 \rightarrow 000000 \\
000011 \rightarrow 000011 \rightarrow 000011 \\
001100 \rightarrow 001100 \rightarrow 001100 \\
001111 \rightarrow 001111 \rightarrow 001111 \\
110000 \rightarrow 111111 \rightarrow 111111 \\
110011 \rightarrow 111001 \rightarrow 111100 \\
111100 \rightarrow 110110 \rightarrow 110011 \\
111111 \rightarrow 110101 \rightarrow 110000
\end{array}
$$

Measure qubits 3,4,5,6 $\longrightarrow$ $|00\rangle + |11\rangle$

Measurement patterns, 0000,0011,1100,1111 (no correction)

The Bit flip code for communication



- Look at the case with X error on Bell pair 12

$$(|01\rangle + |10\rangle)(|00\rangle + |11\rangle)(|00\rangle + |11\rangle)$$

$$\begin{aligned}
010000 &\rightarrow 010000 \rightarrow 010101 \\
010011 &\rightarrow 010011 \rightarrow 010110 \\
011100 &\rightarrow 011100 \rightarrow 011001 \\
011111 &\rightarrow 011111 \rightarrow 011010 \\
100000 &\rightarrow 101010 \rightarrow 101010 \\
100011 &\rightarrow 101001 \rightarrow 101001 \\
101100 &\rightarrow 100110 \rightarrow 100110 \\
101111 &\rightarrow 100101 \rightarrow 100101
\end{aligned}$$

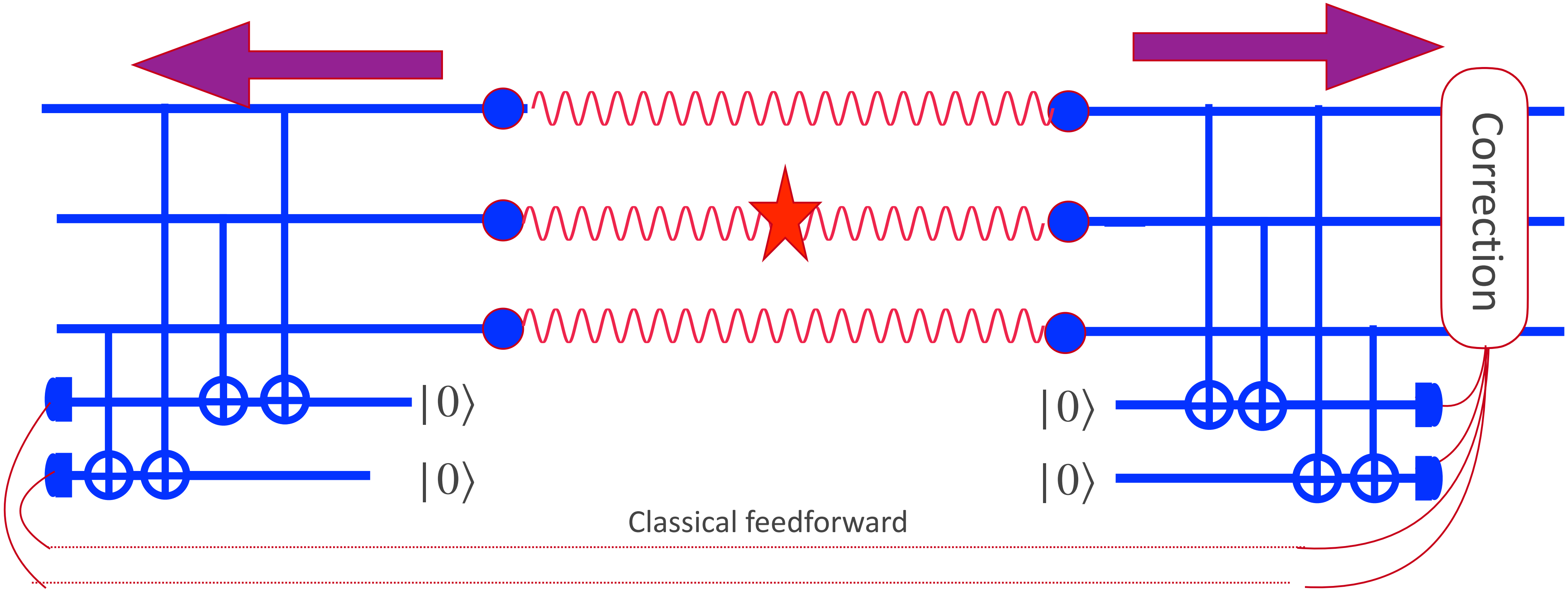
Measure qubits 3,4,5,6

$$|01\rangle + |10\rangle \xrightarrow{X} |00\rangle + |11\rangle$$

Measurement patterns: 0101,0110,1001,1010

The Bit flip code for communication

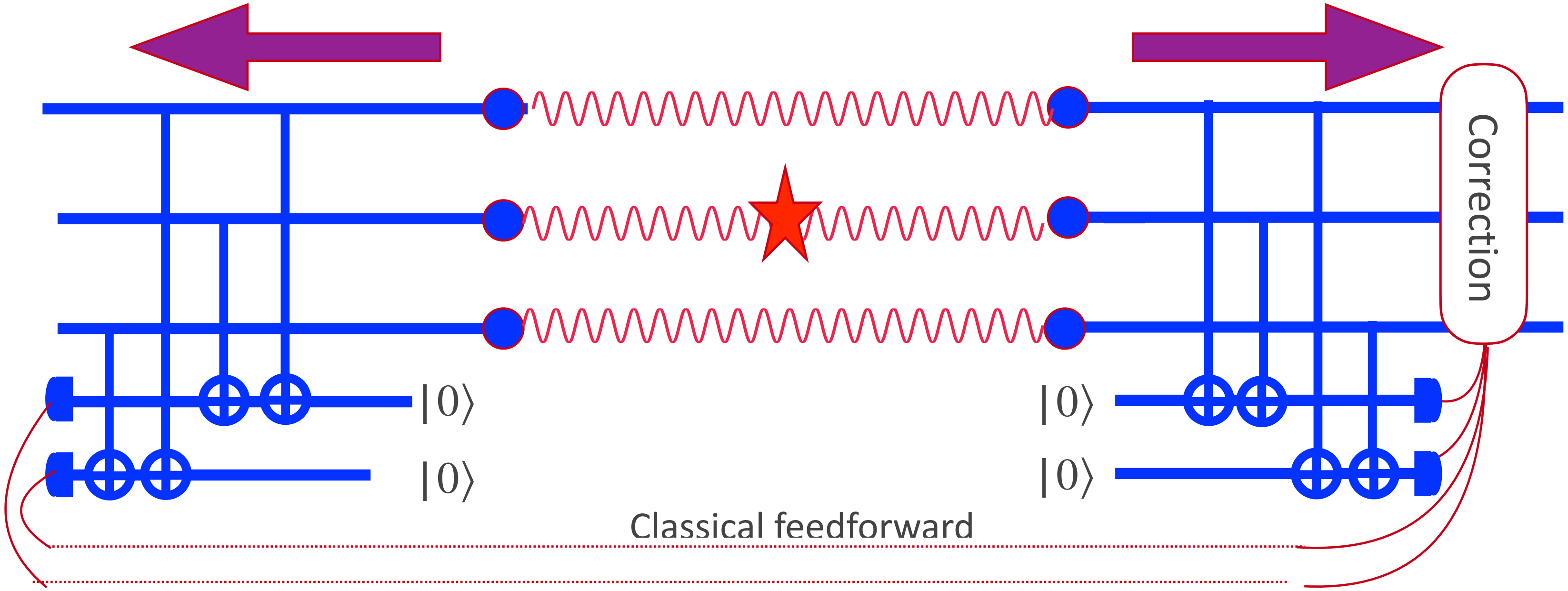
- How do we get this to work for communication?



- Generates an encoded Bell state of the form

$$|\Phi_+\rangle_L = |000\rangle_A|000\rangle_B + |111\rangle_A|111\rangle_B$$

The Pauli frame



- This is an interesting concept in quantum error correction. Our measurements tell us what corrections we need to do (eg X, Z, …)
- We do not always need to do the corrections immediately. Can track them and do it later (the Pauli frame)

Random Errors

- Let p be the probability that $|\Phi_+\rangle$ has a bit flip. We can write the resulting state as $\rho_i = (1-p)|\Phi_+\rangle\langle\Phi_+| + p|\Psi_+\rangle\langle\Psi_+|$
- For our bit flip code with an initial state $\rho_T = \rho_1 \otimes \rho_2 \otimes \rho_3$ we can express the state as

$$\begin{aligned}\rho_T = &(1-p)^3|\Phi_+\rangle\langle\Phi_+| \otimes |\Phi_+\rangle\langle\Phi_+| \otimes |\Phi_+\rangle\langle\Phi_+| \\ &+p(1-p)^2|\Phi_+\rangle\langle\Phi_+| \otimes |\Phi_+\rangle\langle\Phi_+| \otimes |\Psi_+\rangle\langle\Psi_+| + \text{cyclic permutations} \\ &+p^2(1-p)|\Phi_+\rangle\langle\Phi_+| \otimes |\Psi_+\rangle\langle\Psi_+| \otimes |\Psi_+\rangle\langle\Psi_+| + \text{cyclic permutations} \\ &+p^3|\Psi_+\rangle\langle\Psi_+| \otimes |\Psi_+\rangle\langle\Psi_+| \otimes |\Psi_+\rangle\langle\Psi_+|\end{aligned}$$

- Our resulting state after quantum error correction is

$$(1-p')|\Phi_+\rangle_L\langle\Phi_+| + p'|\Psi_+\rangle_L\langle\Psi_+| \quad \text{with} \quad p' = 3p^2 - 2p^3$$

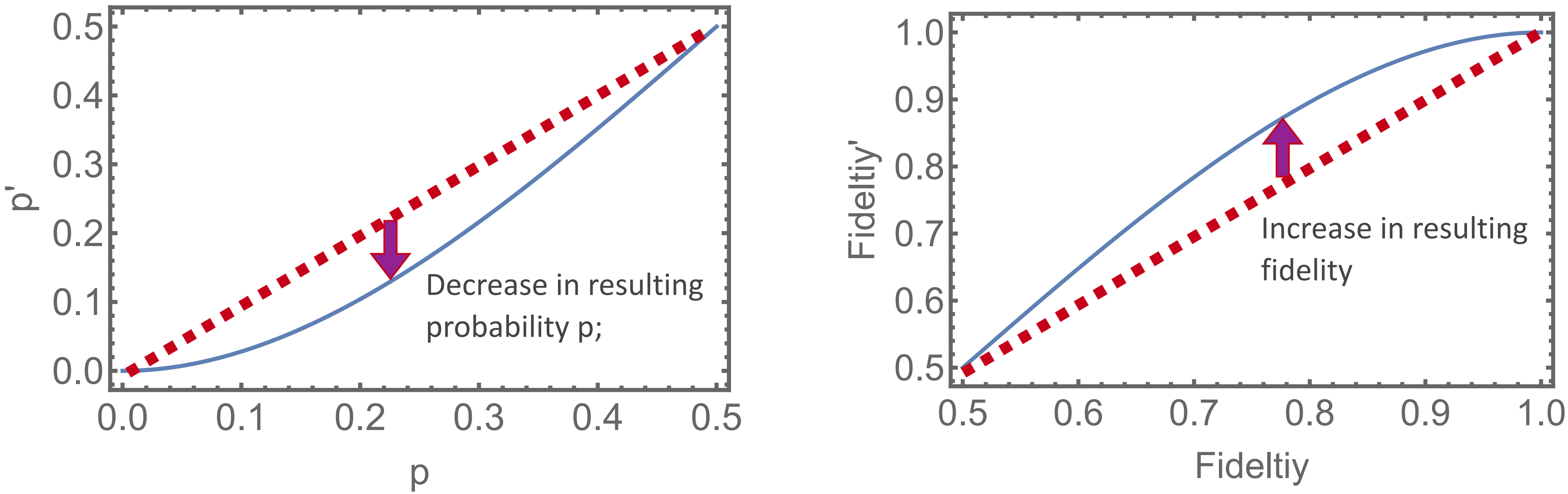
$|\Phi_+\rangle_L = |000\rangle_A|000\rangle_B + |111\rangle_A|111\rangle_B$

$|\Psi_+\rangle_L = |000\rangle_A|111\rangle_B + |111\rangle_A|000\rangle_B$

$p' < p$ for $p < 1/2$

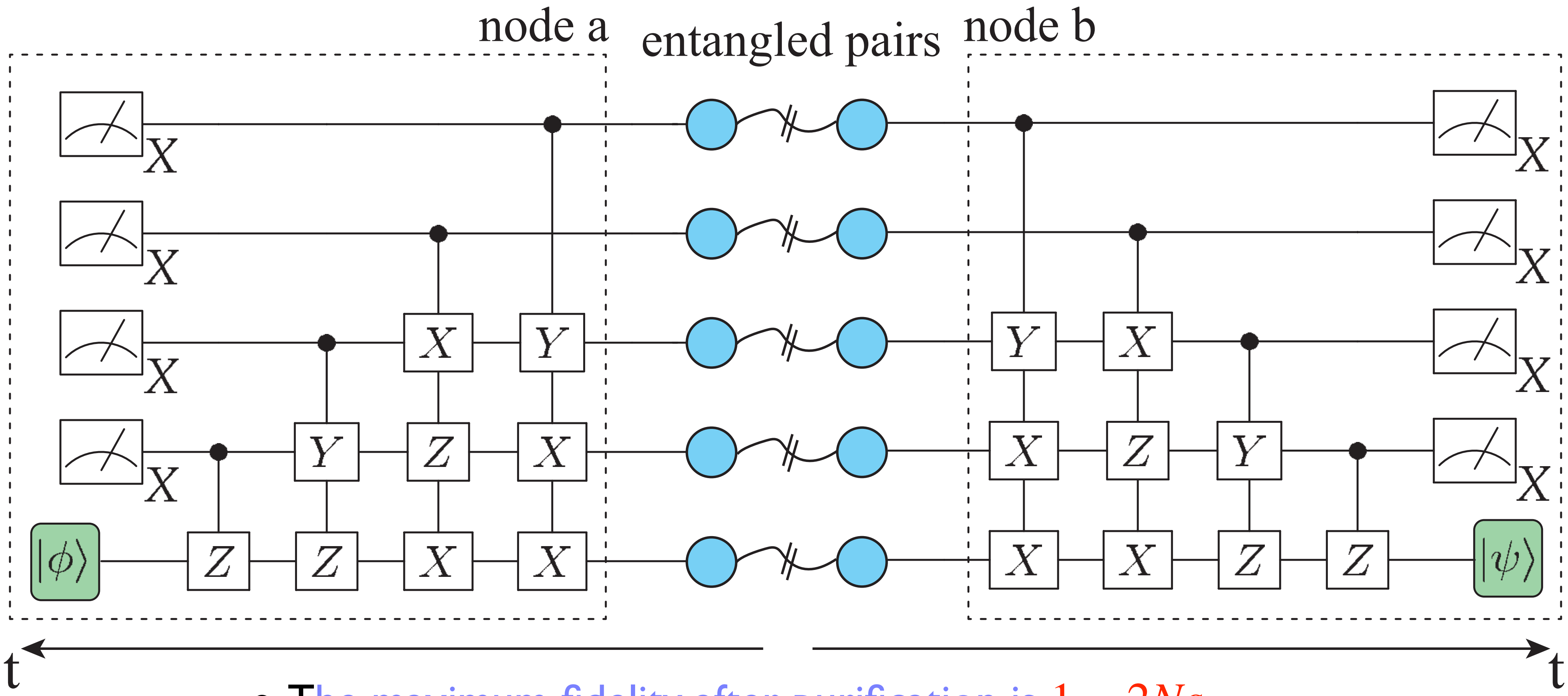
Correcting Random errors

- Now we can see the effect of using the simple bit flip code on our three imperfect Bell pairs



$$F_i = \langle \Phi_+ |_L \rho_i | \Phi_+ \rangle_L$$

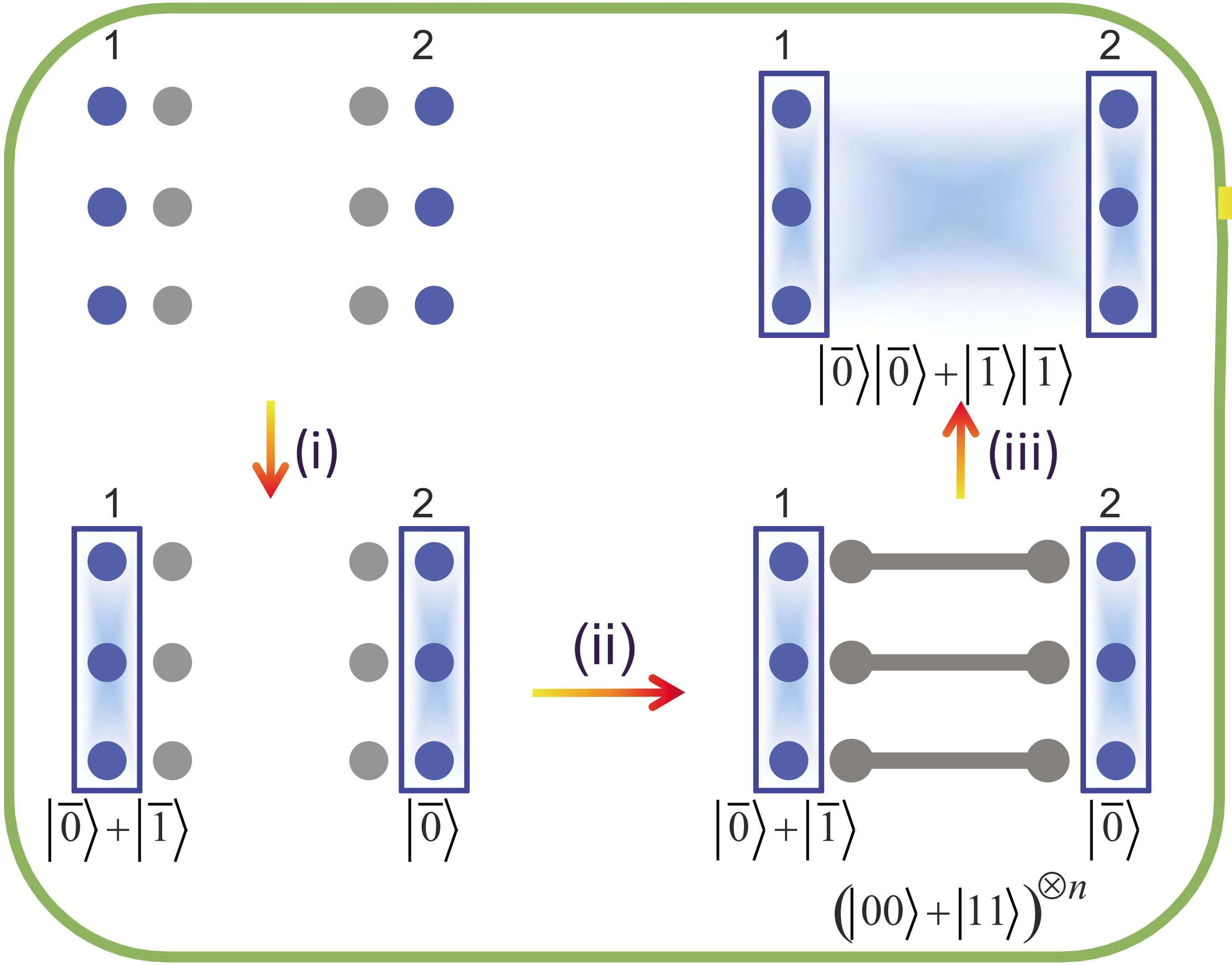
5 Qubit example for a general error (X,Z or XZ)



- The maximum fidelity after purification is $1 - 2N\epsilon_{\text{gate}}$

H. Aschauer, PhD. thesis, Ludwig Maximilians Universität, München, 2004

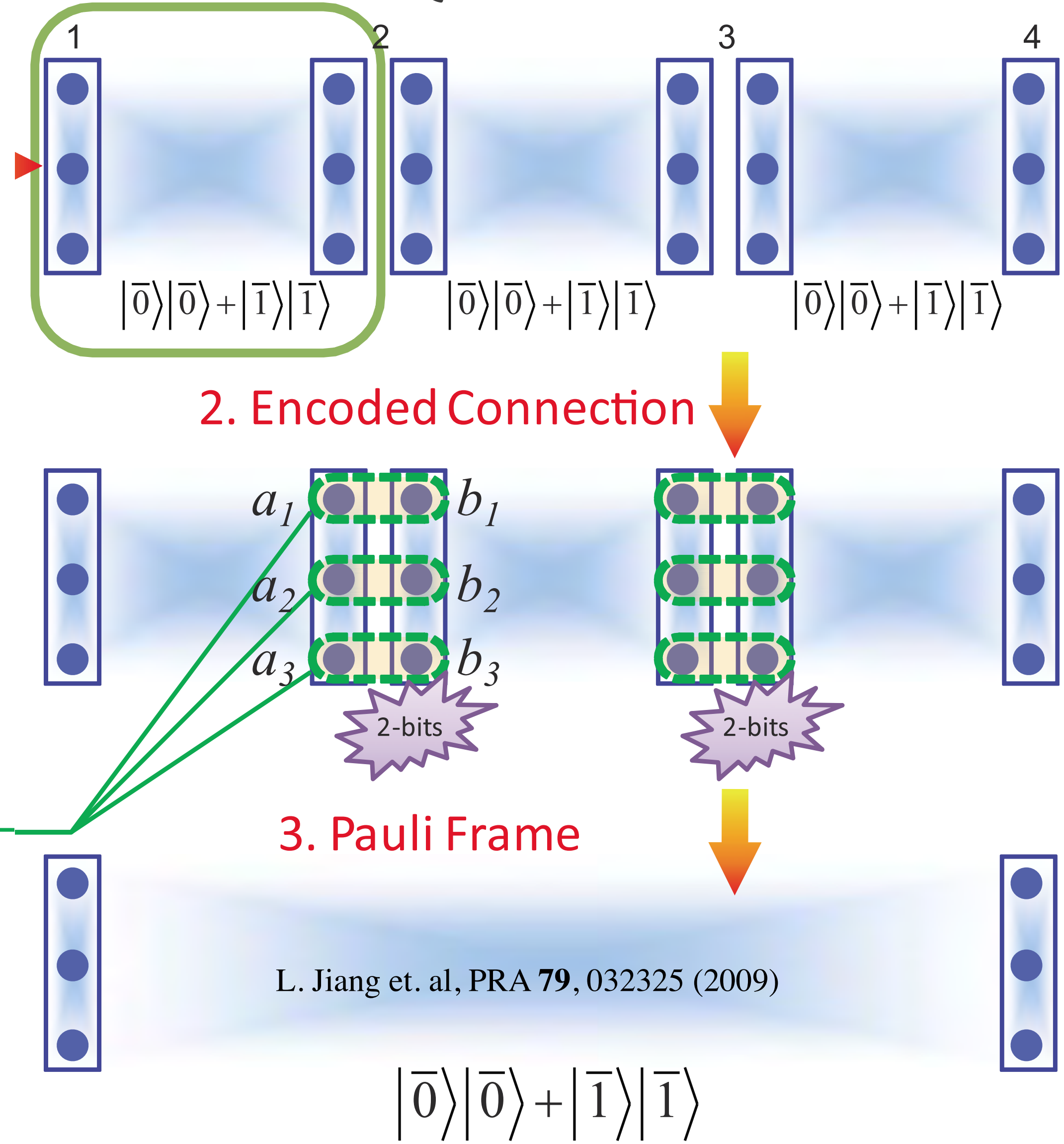
How does this help?

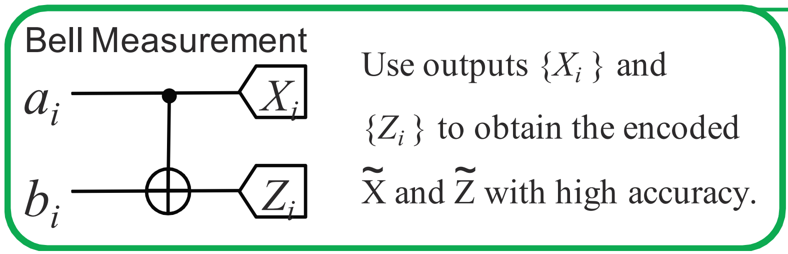


L. Jiang et. al, PRA **79**, 032325 (2009)

Second Generation QR

- Quantum error correction not only allows us to generate logical Bell states between nodes
- We can do logical entanglement swapping operations extending our range.
- Pauli frame means we do not need to wait for the classical comminicaiton





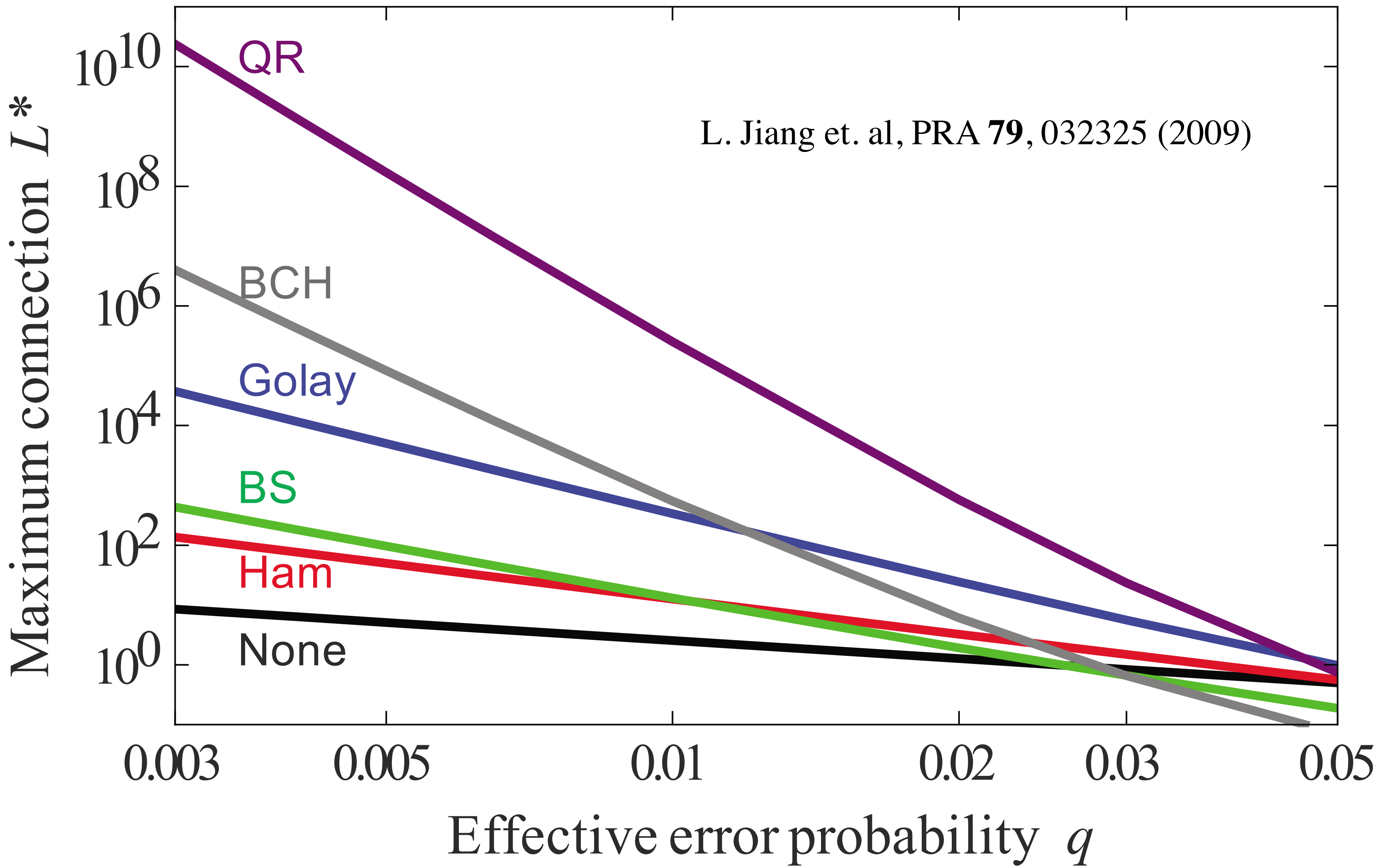
Local resources and maximum communication distance

Name	Code $[[n,k,2t+1]]$	Resources (qubits/station)	Distance (km)
No encoding			180
Repetition-3	[3,1,3]	4	1.0×10^4
Repetition-5	[5,1,5]	18	1.0×10^6
Hamming	[[7,1,3]]	30	1.4×10^3
Bacon-Shor	[[25,1,5]]	42	4.3×10^3
Golay	[[23,1,7]]	150	3.7×10^5
BCH	[[127,29,15]]	138	4.0×10^7
QR	[[103,1,19]]		2.4×10^{11}

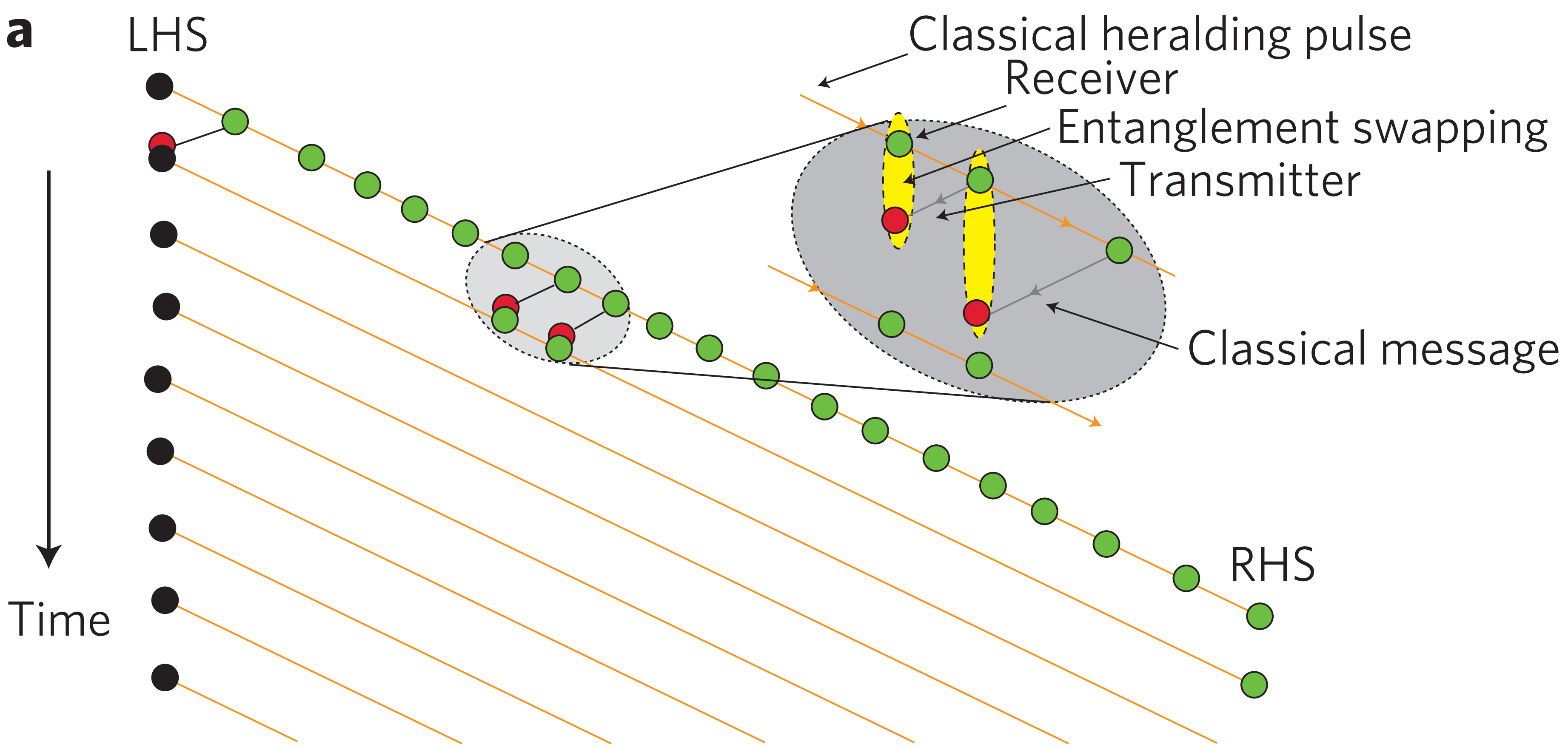
$q=0.3\%$, $F^*=0.95$, and $l_0=10$ km.

L. Jiang et. al, PRA **79**, 032325 (2009)

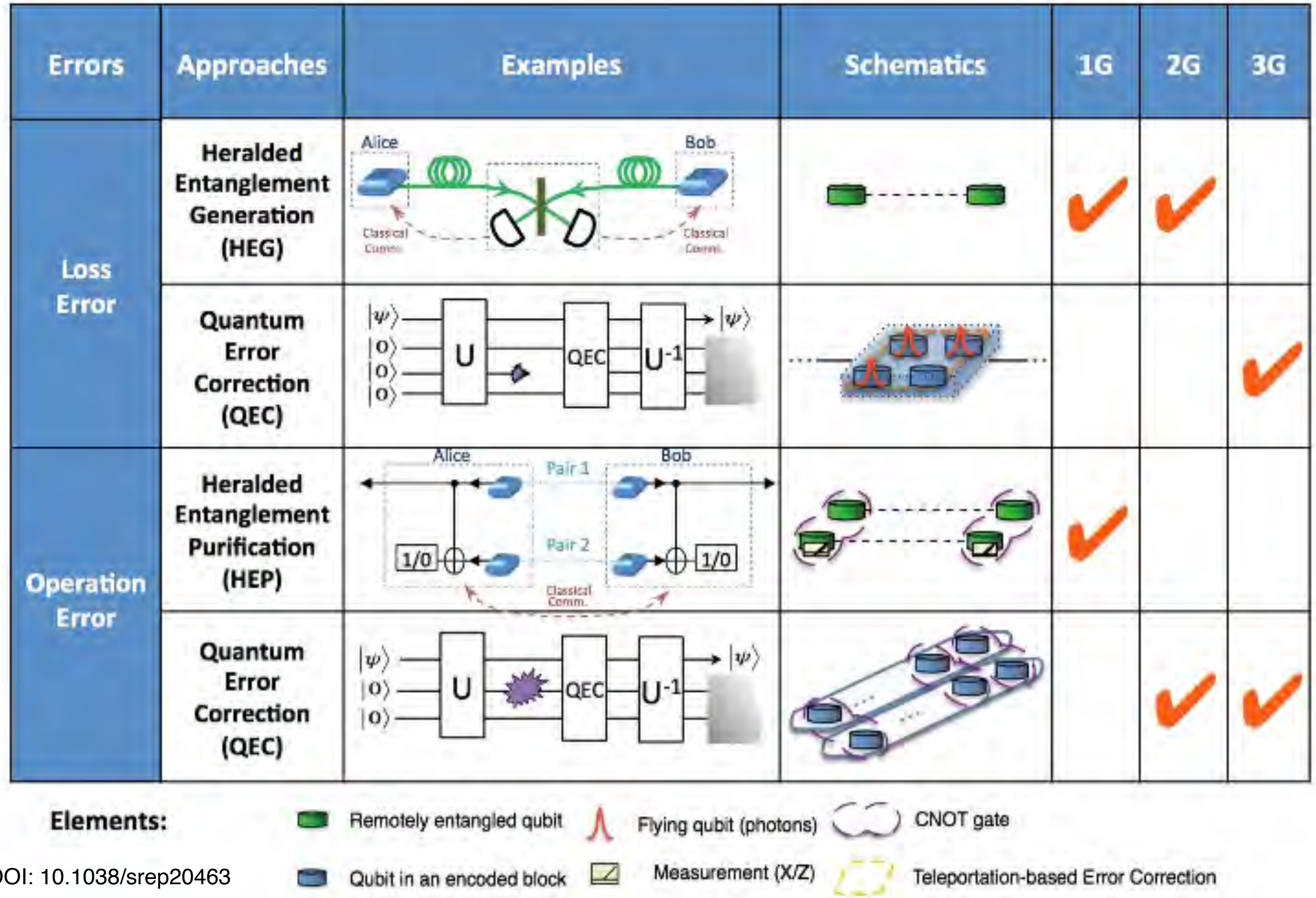
Local resources and maximum communication distance



A little trick



Quantum Repeaters



Scientific **Reports** | 6:20463 | DOI: 10.1038/srep20463

Figure 1. A list of methods to correct loss and operation errors. Depending on the methods used to correct the errors, QRs are categorized into three generations.

Quantum Repeaters

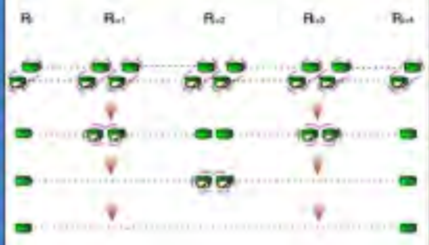
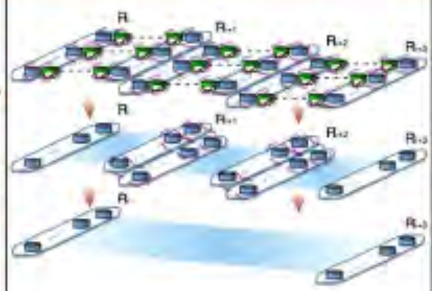
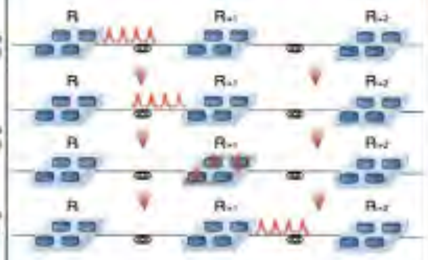
	First Generation QR	Second Generation QR	Third Generation QR
Schematic Architecture 			
Loss Error	HEG (two-way signaling)	HEG (two-way signaling)	QEC (one-way signaling)
Operation Error	HEP (two-way signaling)	QEC (one-way signaling)	QEC (one-way signaling)
Procedure	1. Create entangled pairs over L_0 between adjacent stations 2. At k-th level, connect two pairs over L_k and extend to $L_{k+1}=2L_k$, followed by HEP. 3. After n nesting levels, obtain high-fidelity pair over $L_{tot}=2^n \times L_0$	1. Prepare encoded states $\|0\rangle_L$ and $\|+\rangle_L$ 2. Use teleportation-based non-local CNOT gates to create encoded Bell pairs between adjacent stations. 3. Connect intermediate stations to create long distance encoded Bell pair	1. Encode information with a block of qubits that are sent through a lossy channel 2. Use QEC to correct both loss and operation errors 3. Relay the encoded information to the next station; and repeat steps 2 & 3.
Characteristic time scale	$\mathrm{Max}(L_{tot}/c, t_0)$	$\mathrm{Max}(L_0/c, t_0)$	t_0
Cost Coefficient (C')	$\mathrm{Poly}(L_{tot})$	$\mathrm{PolyLog}(L_{tot})$	$\mathrm{PolyLog}(L_{tot})$

Figure 2. Comparison of three generations of QRs.

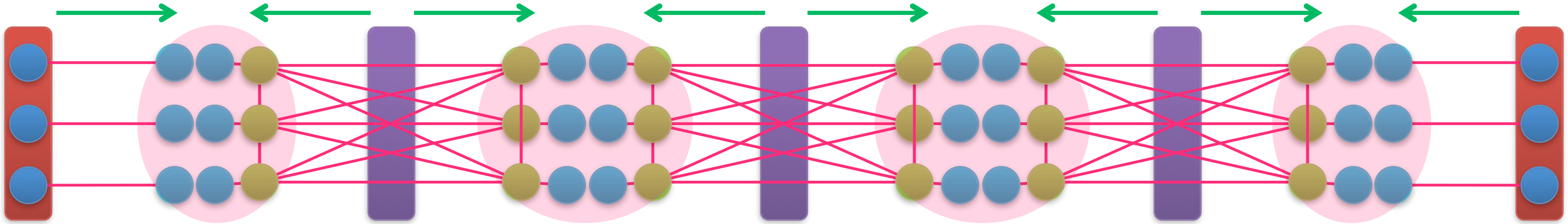
Scientific **Reports** | 6:20463 | DOI: 10.1038/srep20463

All photonic quantum repeaters

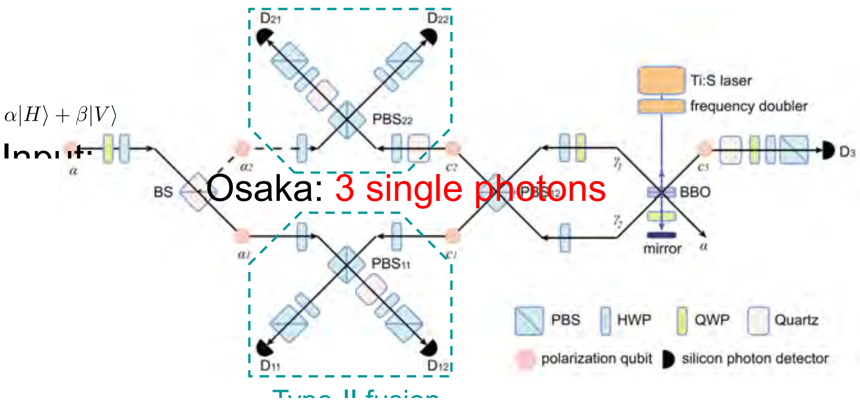
- **Repeaters do not need quantum memories and can use only photonic components.**

The scheme only requires
- Linear optical elements
- Single-photon sources
- Photon detectors
- Fast active feedforward techniques

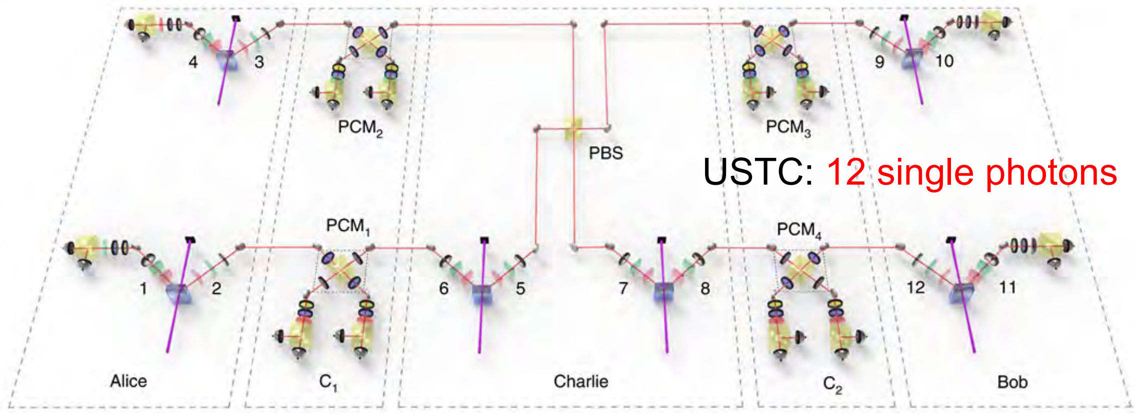
Koji Azuma *et al.*, Nat. Commun. **6**, 6787 (2015).



Proof-of-principle experiments:



Y. Hasegawa *et al.*, Nat. Commun. **10**, 378 (2019).



Z.-D. Li *et al.*, Nat. Photon. **13**, 644 (2019).

Notation

- **A repeater is an electronic device that receives a signal and retransmits it**
- We need to think about the notation from the classical work in terms of all photonic devices
- We have regenerators
  - 1R-regenerators - reamplification
  - 2R regenerators - reamplification and reshaping
  - 3R regenerators - reamplification, reshaping, and retiming

- **So 2nd and 3nd generation repeaters should be called regenerators (not repeaters) if all photonic**

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